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Scalar σ meson via chiral and crossing dynamics

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Abstract. We show that the non-strange scalar σ meson, as now reported in the 1996 PDG tables, is a natural consequence of crossing symmetry as well as chiral dynamics for both strong interaction low energy $\pi\pi$ scattering and also $K \to 2\pi$ weak decays.

1 Introduction

The 1996 Particle Data Group (PDG) tables [1] now includes a broad non-strange I=0 scalar σ resonance referred to as f_0 (400-1200). This is based in part on the Törnqvist-Roos [2] re-analysis of low energy $\pi\pi$ scattering, finding a broad non-strange σ meson in the 400-900 MeV region with pole position $\sqrt{s_0}$ = 0.470 - i 0.250 GeV. Several later comments in PRL [3–5] all stress the importance of rejecting [3] or confirming [4, 5] the above Törnqvist-Roos [2] σ meson analysis based on (t-channel) crossing symmetry of this $\pi\pi$ process.

In this brief report we offer such a σ meson-inspired crossing symmetry model in support of [2, 4, 5] based on chiral dynamics for strong interaction $\pi\pi$ scattering (Sect. 2). This in turn supports the recent s-wave $\pi\pi$ phase shift analyses [6] in Sect. 3 using a negative background phase obtaining a broad σ resonance in the 535-650 MeV mass region. This is more in line with the prior analysis of [5] and with the dynamically generated quark-level linear σ model (LσM) theory of [7] predicting $m_\sigma \approx 650$ MeV. Section 4 looks instead at processes involving two final-state pions where crossing symmetry plays no role, such as for the DM2 experiment [8] J/$\Psi \to \omega\pi\pi$ and for πN $\to \pi\pi$N polarization measurements [9]. Section 5 extends the prior crossing-symmetric strong interaction chiral dynamics to the non-leptonic weak interaction ΔI = $\frac{1}{2}$ decays $K^\circ \to 2\pi$. We give our conclusions in Sect. 6.

2 Strong interactions, crossing symmetry and the σ meson

It has long been understood [10–12] that the non-strange isospin I=0 σ meson is the chiral partner of the I=1 pion. In fact Gell-Mann-Lévy's [10, 11] nucleon-level LσM requires the meson-meson couplings to satisfy (with $f_\pi \approx$ 93 MeV)

$$g_{\sigma\pi\pi} = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi} = \lambda f_\pi \ , \tag{1}$$

where $g_{\sigma\pi\pi}$ and λ are the cubic and quartic meson couplings respectively. On the other hand, the σ meson pole for the $\pi\pi$ scattering amplitude at the soft point $s = m_\pi^2$ using (1) becomes

$$M_{\pi\pi}^{\sigma\text{pole}} = \frac{2g_{\sigma\pi\pi}^2}{s - m_\sigma^2} \to \frac{2g_{\sigma\pi\pi}^2}{m_\pi^2 - m_\sigma^2} = -\lambda = -M_{\pi\pi}^{\text{contact}} \ . \tag{2}$$

The complete tree-level LσM $\pi\pi$ amplitude is the sum of the quartic contact amplitude λ plus σ poles added in a *crossing symmetric* fashion from the s, t and u-channels. Using the chiral symmetry soft-pion limit (2) combined with the (non-soft) Mandelstam relation s + t + u $= 4m_\pi^2$, the lead λ contact $\pi\pi$ amplitude approximately cancels [11]. Not surprisingly, the resulting net $\pi^a\pi^b \to \pi^c\pi^d$ amplitude in the LσM is the low energy model-independent Weinberg amplitude [13].

$$M_{\pi\pi} = \frac{s - m_\pi^2}{f_\pi^2}\ \delta^{ab}\delta^{cd} + \frac{t - m_\pi^2}{f_\pi^2}\ \delta^{ac}\delta^{bd} + \frac{u - m_\pi^2}{f_\pi^2}\ \delta^{ad}\delta^{bc} \ , \tag{3}$$

due to partial conservation of axial currents (PCAC) applied crossing-consistently to all three s, t, u-channels. Recall that the underlying PCAC identity $\partial A^i = f_\pi m_\pi^2 \phi_\pi^i$, upon which the Weinberg crossing-symmetric PCAC relation (3) is based, was originally obtained from the LσM lagrangian [10, 11].

Although the above (LσM) Weinberg PCAC $\pi\pi$ amplitude (3) predicts an s-wave I=0 scattering length [13] $a_{\pi\pi}^{(0)} = 7m_\pi/32\pi f_\pi^2 \approx 0.16\ m_\pi^{-1}$ which is ~30% less than

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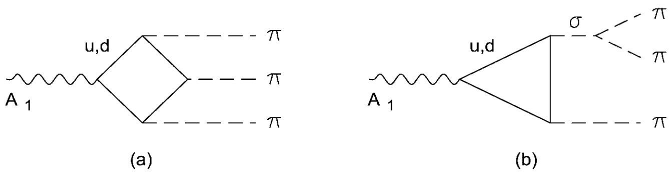

Fig. 1. **a** Quark box, **b** quark triangle graphs for $A_1 \to 3\pi$

first obtained from $K_{\ell 4}$ data [14], more precise experiments are now under consideration. Moreover a simple chiral-breaking scattering-length correction $\Delta a^0_{\pi\pi}$ follows from the LσM using a Weinberg-like crossing-symmetric form [15]

$$M^{abcd}_{\pi\pi} = A(s,t,u)\delta^{ab}\delta^{cd} + A(t,s,u)\delta^{ac}\delta^{bd} + A(u,t,s)\delta^{ad}\delta^{bc} \,, \quad (4)$$

$$A^{L\sigma M}(s,t,u) = -2\lambda \left[1 - \frac{2\lambda f_\pi^2}{m_\sigma^2 - s}\right] = \left(\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s}\right)\left(\frac{s - m_\pi^2}{f_\pi^2}\right) \,, \quad (5)$$

where the LσM Eq. (1) has been used to obtain the second form of (5). Then the I=0 s-channel amplitude 3A(s, t, u) + A(t, s, u) + A(u, t, s) predicts the s-wave scattering length at s $= 4m_\pi^2$, t = u = 0 using the LσM amplitude (5) with $\varepsilon = m_\pi^2/m_\sigma^2 \approx 0.046$ for the LσM mass [7] $m_\sigma \approx$ 650 MeV:

$$a^{(0)}_{\pi\pi}|_{\mathrm{L}\sigma\mathrm{M}} \approx \left(\frac{7+\varepsilon}{1-4\varepsilon}\right)\frac{m_\pi}{32\pi f_\pi^2} \approx (1.23)\frac{7m_\pi}{32\pi f_\pi^2} \approx 0.20 m_\pi^{-1} \,. \quad (6)$$

This simple 23% LσM enhancement of the Weinberg PCAC prediction [13] agrees in magnitude with the much more complicated one-loop order chiral perturbation theory approach [16] which also predicts an s-wave scattering length correction of order $\Delta a^0_{\pi\pi} \sim 0.04 m_\pi^{-1}$. This indirectly supports a $\sigma(650)$ scalar meson mass scale as used in (6).

The above exact (chiral symmetry) cancellation, due to (1) and (2) has been extended to final-state pionic processes $A_1 \to \pi(\pi\pi)_{s-\mathrm{wave}}$ [17], $\gamma\gamma \to 2\pi^0$ [18] and $\pi^- p \to \pi^-\pi^+ n$. In all of these cases the above LσM chiral cancellation is simulated by a (non-strange) quark box – quark triangle cancellation due to the Dirac-matrix identity [17, 18]

$$\frac{1}{\gamma.p - m}\, 2m\gamma_5 \,\frac{1}{\gamma.p - m} = -\gamma_5 \,\frac{1}{\gamma.p - m} - \frac{1}{\gamma.p - m}\,\gamma_5 \,, \quad (7)$$

combined with the quark-level Goldberger relation (GTR) $f_\pi g_{\pi qq} = m_q$ and the LσM meson couplings in (1).

Then the u, d quark box graph in Fig. 1a for $A_1 \to 3\pi$ in the chiral limit exactly cancels the quark triangle graph of Fig. 1b coupled to the σ meson because of the GTR and the LσM chiral meson identity (1) along with the minus signs on the right-hand-side (rhs) of (7):

$$M^{\mathrm{box}}_{A_1 3\pi} + M^{\mathrm{tri}}_{A_1 3\pi} \to -\frac{1}{f_\pi} M(A_1 \to \sigma\pi) + \frac{1}{f_\pi} M(A_1 \to \sigma\pi) = 0 \,. \quad (8)$$

This soft pion theorem [17] in (8) is compatible with the PDG tables [1] listing the decay rate $\Gamma[A_1 \to \pi(\pi\pi)_{sw}] = 1 \pm 1$ MeV.

Similarly, the $\gamma\gamma \to 2\pi^0$ quark box graph suppresses the quark triangle σ resonance graph in the 700 MeV region, also compatible with $\gamma\gamma \to 2\pi^0$ cross section data [18]. Finally, the peripheral pion in $\pi^- p \to \pi^-\pi^+ n$ sets up an analogous $\pi\pi$ or quark box – quark triangle s-wave soft pion cancellation which completely suppresses any such σ resonance – also an experimental fact for $\pi^- p \to \pi^-\pi^+ n$.

3 $\pi\pi$ phase shifts

The above approximate (chiral) cancellation in $\pi\pi \to \pi\pi$, $A_1 \to 3\pi, \gamma\gamma \to 2\pi^0$ and $\pi^- p \to \pi^-\pi^+ n$ amplitudes and in data lends indirect support to the analyses of [2, 4, 5]. Reference [3] claims instead that the I=0 and I=2 $\pi\pi$ phase shifts require t-channel forces due to "exotic", crossing-asymmetric resonances in the I=$\frac{3}{2}$ and 2 cross-channels rather than due a broad low-mass scalar σ meson (in the s-channel). We suggest that this latter picture in [3] does not take account of the crossing-symmetric extent of the chiral $\pi\pi$ forces in all three s, t and u-channels, leading to the above approximate chiral cancellation.

Specifically the recent $\pi\pi$ phase shift analyses in [6] use a negative background phase approach compatible with unitarity. This background phase has a hard core of size $r_c \approx 0.63\ fm$ (the pion charged radius) such that $\delta^{BG} = -p_\pi^{\mathrm{CM}} r_c$. Combining this background phase with the observed $\pi\pi$ phase shifts (e.g., of CERN-Munich or Cason et al.), the new I=0 phase shift goes through 90° resonance in the range 535-650 MeV, while the I=2 phase shift does not resonate but remains negative as observed. [6] justify this background phase approach because of the "compensating $\lambda\phi^4$ contact (LσM) interaction". From our Sect. 2 we rephrase this as due to the *crossing symmetric* LσM chiral approximate cancellation [11] which recovers Weinberg's [13] PCAC $\pi\pi$ amplitude in our (3).

Then [6] choose a slightly model-dependent form factor F(s) (designed to fit the lower energy region below 400 MeV) along with the best-fitted $\sigma \to \pi\pi$ effective coupling (double the LσM field theory coupling (1)). This gives the resonant σ width [6]

$$\Gamma_R(s) = \frac{p_\pi^{\mathrm{CM}}}{8\pi s}[g_R F(s)]^2 \approx 340 \text{ MeV} \quad \text{at} \quad \sqrt{s}_R \approx 600 \text{ MeV}, \quad g_R \approx 3.6 \text{ GeV} \,, \quad (9)$$

for $p_\pi^{\mathrm{CM}} = \sqrt{s/4 - m_\pi^2} \approx 260$ MeV. However, the decay width in (9) accounts only for $\sigma \to \pi^+\pi^-$ decay. To include as well the $\sigma \to \pi^0\pi^0$ decay mode, one must scale up (9) by a factor of 3/2:

$$\Gamma_{\sigma\to 2\pi} = \frac{3}{2}\Gamma_R(s) \approx 510 \text{ MeV} , \quad (10)$$

not incompatible with [1, 2, 5] but still slightly below Weinberg's recent mended chiral symmetry (MCS) prediction [19]

$$\Gamma^{\text{MCS}}_{\sigma\to 2\pi} = \frac{9}{2}\Gamma_\rho \approx 680 \text{ MeV} , \quad (11a)$$

or the LσM decay width [15]

$$\Gamma^{L\sigma M}_{\sigma\to 2\pi} = \frac{3}{2}\ \frac{p^{\text{CM}}_\pi}{8\pi}\ \frac{(2g_{\sigma\pi\pi})^2}{m^2_\sigma} \approx 580 \text{ MeV} , \quad (11b)$$

for $m_\sigma \approx 600$ MeV. Note too that the best fit $\sigma \to \pi^+\pi^-$ effective coupling in [6] of 3.60 GeV is close to the LσM value in (1) at $m^R_\sigma \approx 600$ MeV:

$$g_R \to 2g_{\sigma\pi\pi} = (m^2_\sigma - m^2_\pi)/f_\pi \approx 3.66 \text{ GeV} . \quad (12)$$

4 Crossing-asymmetric determinations of σ (600-750)

With hindsight, the clearest way to measure the $\sigma \to \pi\pi$ signal is to avoid $\pi\pi \to \pi\pi, \gamma\gamma \to 2\pi^\circ, \pi^- p \to \pi^-\pi^+ n$ scatterings or $A_1 \to \pi(\pi\pi)_{sw}$ decay, since these processes are always plagued by the $\pi\pi$ exact chiral cancellation in (2) or an underlying quark box – triangle cancellation due to (7) as in (8). First consider the 1989 DM2 experiment [8] $J/\Psi \to \omega\pi\pi$. Their Fig. 13 fits of the $\pi^+\pi^-$ and $\pi^\circ\pi^\circ$ distributions clearly show the known non-strange narrow $f_2(1270)$ resonance along with a broad $\sigma(500)$ "bump" (both bumps are non-strange and the accompanying ω is 97% non-strange). Moreover, DM2 measured the (low mass) σ width as [8]

$$\Gamma^{\text{DM2}}_{\sigma\to\pi\pi} = 494 \pm 58 \text{ MeV} , \quad (13)$$

very close to the modified [6] σ width fit of 510 MeV in (10).

Finally, this Fig. 13 of DM2 [8] clearly shows that the nearby $f_0(980)$ bump in the $\pi\pi$ distribution is only a "pimple" by comparison. This suggests that the observed [1] $f_0(980) \to \pi\pi$ decay mode proceeds via a small $\sigma - f_0$ mixing angle and that $f_0(980)$ is primarily an $\bar{s}s$ meson, compatible with the analyses of [2, 20]. However, such a conclusion is not compatible with the $\overline{qq}qq$ or $K\overline{K}$ molecule studies noted in [3].

Lastly, polarization measurements are also immune to the (spinless) approximate chiral cancellation [11] in $\pi\pi \to \pi\pi$. This detailed polarization analysis of [9] approximately obtains the ρ (770) mass and 150 MeV decay width. While the resulting σ mass of 750 MeV is well within the range reported in the 1996 PDG [1] and closer to the σ mass earlier extracted from $\pi\pi \to K\overline{K}$ studies in [21], the inferred σ width of $\Gamma_\sigma \sim 200 - 300$ MeV in [9] is much narrower than reported in [1, 2, 8, 21] or in our above analysis.

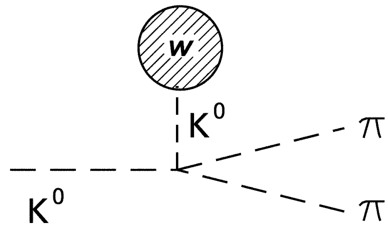

Fig. 2. ΔI=1/2 t-channel K° tadpole graph for $K^\circ \to 2\pi$

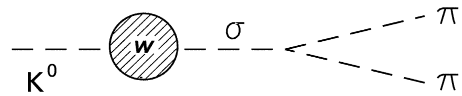

Fig. 3. ΔI=1/2 s-channel σ pole graph for $K^\circ \to 2\pi$

5 $K^\circ \to 2\pi$ weak decays and the σ(600-700) meson

To show that the σ(600-700) scalar meson also arises with chiral crossing-symmetric weak forces, we consider the ΔI=1/2 – dominant $K^\circ \to 2\pi$ decays. To manifest such a ΔI=1/2 transition, we first consider the virtual K° $I = \frac{1}{2}$ meson t-channel tadpole graph of Fig. 2. Here the weak tadpole transition $< 0|H_w|K^\circ >$ clearly selects out the ΔI=1/2 part of the parity-violating component of H_w, while the adjoining strong interaction $K^\circ\overline{K}^\circ \to \pi\pi$ is the kaon analogue of the t-channel $\pi\pi \to \pi\pi$, with Weinberg-type PCAC [22] amplitude $(t - m^2_\pi)/2f^2_\pi$ for $t = (p_K - 0)^2 = m^2_K$. Then the ΔI=1/2 amplitude magnitude is [23]

$$|<\pi\pi|H_w|K^\circ>| = \frac{|<0|H_w|K^\circ>|}{2f^2_\pi}(1 - m^2_\pi/m^2_K) . \quad (14)$$

A crossed version of this ΔI=1/2 transition (14) is due to the s-channel I=0 σ meson pole graph of Fig. 3 at $s = m^2_K$ [24]. This leads to the ΔI=1/2 amplitude magnitude

$$|<\pi\pi|H_w|K^\circ>| = |<\pi\pi|\sigma> \frac{1}{m^2_K - m^2_\sigma + im_\sigma\Gamma_\sigma} <\sigma|H_w|K^\circ>| . \quad (15a)$$

Applying chiral symmetry $<\sigma|H_w|K^\circ> = <\pi^\circ|H_w|K^\circ>$ (converting the former parity-violating to the latter parity-conserving transition) along with the LσM values $|<\pi\pi|\sigma>| = m^2_\sigma/f_\pi$ from (1) and $\Gamma_\sigma \approx m_\sigma$ to (15a), one sees that the σ mass scale cancels out of (15a), yielding [25]

$$|<\pi\pi|H_w|K^\circ>| \approx |<\pi^\circ|H_w|K^\circ>/f_\pi| . \quad (15b)$$

Not only has (15b) been derived by other chiral methods [26], but (15b) also is equivalent to (14) in the $m_\pi = 0$ chiral limit because weak chirality $[Q, H_w] = -[Q_5, H_w]$ for V-A weak currents and PCAC clearly require $|<\pi^\circ|H_w|K^\circ>| \approx |<0|H_w|K^\circ>/2f_\pi|$, as needed.

Thus, we see that the existence of an I=0 scalar σ meson below 1 GeV manifests crossing symmetry (from the t to the s-channel) for the dominant ΔI=1/2 equivalent amplitudes (14) and (15b). Further use of the quark model and the GIM mechanism [27] converts the $K^{\circ}_{2\pi}$ amplitudes in (14) or (15b) to the scale [23] 24×10^{-8} GeV, close to the observed $K^{\circ}_{2\pi}$ amplitudes [1].

While the ΔI=1/2 $K^{\circ} \to 2\pi$ decays are controlled by the tadpole diagram in Fig. 2 (similar to ΔI=1 Coleman-Glashow tadpole for electromagnetic (em) mass splittings [28, 29]), the smaller ΔI=3/2 $K^{+} \to 2\pi$ amplitude is in fact suppressed by "exotic" I=3/2 meson cross-channel Regge trajectories [30] (in a manner similar to the I=2 cross-channel exotic Regge exchange for the $\pi^{+}-\pi^{\circ}$ em mass difference [31]). This latter duality nature of crossing symmetry for exotic I=3/2 and I=2 channels was invoked in [3] to *reject* the low mass σ meson scheme reported in the 1996 PDG tables [1] based in part on the data analysis of [2]. That is, for exotic I=2 and I=3/2 (t-channel) dual exchanges, the dynamical dispersion relations thus generated are unsubtracted, so that one can then directly estimate the observed ΔI=2 em mass differences [32] and also the ΔI=3/2 weak $K^{+}_{2\pi}$ decay amplitude [33]. However, for I=1 and I=1/2 dual exchanges, the resulting dispersion relations are once-subtracted, with subtraction constants corresponding to contact ΔI=1 and ΔI=1/2 tadpole diagrams for em and weak transitions, respectively. Contrary to [3], we instead suggest that these duality pictures for exotic I=3/2 and I=2 channels of [30, 31] in fact help *support* the existence of the I=0 chiral σ meson in [2, 4–7].

6 Summary

We have studied both strong and weak interactions involving two final-state pions at low energy, using chiral and crossing symmetry to reaffirm the existence of the low-mass I=0 scalar σ meson below 1 GeV. This supports the recent phenomenological data analyses in [2, 4–6] and the quark-level linear σ model [LσM] theory of [7].

In Sect. 2 we focussed on $\pi\pi$ scattering and the crossing symmetry approximate chiral cancellation [11] in the LσM and its extension to the quark box – quark triangle soft pion cancellation [17, 18]. Such chiral cancellations in $\pi\pi \to \pi\pi, A_1 \to 3\pi, \gamma\gamma \to 2\pi^0, \pi^- p \to \pi^-\pi^+ n$ in turn suppress the appearance of the σ(600-700) meson. Then in Sect. 3 we supported the recent re-analyses [6] of $\pi\pi$ phase shift data invoking a negative background phase. This led to an I=0 σ meson in the 535-650 MeV region, but with a broader width $\Gamma_\sigma \sim 500$ MeV than found in [6] (but not incompatible with the 1996 PDG σ width [1]).

In Sect. 4 we briefly reviewed two different crossing-asymmetric determinations of the I=0 σ(600-750) which circumvent the above crossing-symmetric approximate chiral suppression of the σ meson. Finally, in Sect. 5 we reviewed how the low mass I=0 σ meson s-channel pole for ΔI=1/2 $K^0 \to 2\pi$ decays is needed to cross over to the t-channel ΔI=1/2 tadpole graph (which in turn fits data). This ΔI=1/2 crossing-symmetry $K \to \pi\pi$ picture was also extended by crossing duality to justify why the (much smaller) ΔI=3/2 $K^{+}_{2\pi}$ decay is controlled by exotic I=3/2 t-channel Regge trajectories [30], while the above I=1/2 dispersion relation has a (tadpole) non-exotic Regge subtraction constant.

To verify that the above chiral scheme in fact favors a ground state broad scalar mass σ (600-700) over the higher and narrower ϵ (1300), we note that

i) even though the σ (600-700) is greatly suppressed by the null soft pion theorems (SPT) of Sect. 2, a positive signal of this suppression is the prominent (SPT) dip in the $\gamma\gamma \to 2\pi^0$ cross section in the 600-700 MeV region but not in the 1300 MeV vicinity [34];

ii) the KEK negative background phase approach of Sect. 3 finds a broad σ (535-650) resonance from $\pi\pi$ phase shifts, but does not recover the ϵ (1300) [6];

iii) although the original nucleon level L σ M [10] finds no (tree order) constraint for the σ mass scale, the quark-level L σ M predicts at one-loop order a nonstrange σ (650), but no ϵ (1300) [7];

iv) the DM2 experiment [8] for $J/\Psi \to \omega\pi\pi$ does observe a broad σ (500) bump (but not an ϵ (1300)) in the $\pi\pi$ distribution;

v) the $K^o \to 2\pi$ amplitude via the σ pole in (15a) generating the observed magnitude [25, 26] $| < \pi\pi|H_w|K^0 > | \sim 26 \times 10^{-8}$ GeV when $|m_K^2 - m^2{}_\sigma| << m_\sigma \Gamma_\sigma$ in the denominator of (15a) (as for a LσM σ (650)), would predict instead a decay rate 60% shy of data using instead an ϵ (1300) and moreover the chiral link between (14) and (15) would then be severed;

vi) the original $q^2\bar{q}^2$ bag model estimate of the nonstrange scalar mass finds [35] $m_\sigma \approx 700$ MeV and not 1300 MeV.

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